

REPORT SD-TR-81-11



Collision Hazard in Space

V. A. CHOBOTOV
Guidance and Control Division
The Aerospace Corporation
El Segundo, CA 90245



25 February 1981

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

Prepared for

SPACE DIVISION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
P.O. Box 92960, Worldway Postal Center
Los Angeles, Calif. 90009

DTIC FILE COPY

81 3 27 117

This final report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract F04701-80-C-0081 with the Space Division, PO Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by L. J. Kulakowski, Engineering Group. Colonel D. F. Shane, Office of Plans (SD/CX), was the project engineer.

This report has been reviewed by the Public Affairs Office (SD/PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Robert L. Dean, Lt Colonel, USAF Director, Mission Planning

FOR THE COMMANDER

James P. Baker, Lt Colonel, USAF Acting Assistant for Plans

10 PEP	ORT DOCUMENTATION PAGE	READ INSTRUCTIONS
1 REPORT AUMBER		BEFORE COMPLETING FORM CCESSION NO. 3. RECIPIENT'S CATALOG NUMBER
	1	97004
SD-TR-81-11	AU-AC	
TITLE (and Subtitle)		5 TYPE OF REPORT & PERIOD COVER
	DD TV GD G	11
COLLISION HAZA	IN SPACE.	Final VOL 1
-	-	TR-Ø081(679Ø)-1 *
7. AUTHOR(s)		P. CONTRACT OR GRANT-NUMBER(*)
V. A. Chobotov	(5) FO4	1701-80-C-0031
9. PERFORMING ORGAN	IZATION NAME AND APPRESS	10. PROGRAM ELEMENT, PROJECT, TAS
The Aerospace	Corporation	
El Segundo, CA		(· 1)
	CE NAME AND ADDRESS	M PEPORT DATE
		25 Feb. 1981
Space Divisior Air Force Syst		19. NUMBER OF PAGES
Los Angeles. (Calif. 90009	65
14 MONITORING AGEN	Y NAME & ADDRESS(II different from Conti	olling Office) 15. SECURITY CLASS. (of this report)
(in)	/)	
	6	Unclassified
		15. DECLASSIFICATION DOWNGRADIN
17. DISTRIBUTION STAT	EMENT (of the abstract entered in Block 20	. If different from Report)
18. SUPPLEMENTARY N	OTES	
19. KEY WORDS (Continu	on reverse side if necessary and identify b	y block number)
Satellites	Space Debris Stati	
Collisions	Operational and De	sign Policies
Orbits		
The statistic and analyzed. orbit are des	Several methods for deter	ng population of objects is presente mining the probability of collision ities for low and geosynchronous

DD FORM 1473

UNCLASSIFIE - C / // CO
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

PREFACE

The funding for this study was in part provided by the following program offices: Long Range Planning, Defense Support Systems, Plans and Systems Architecture - Satellite Position Management, Integration Directorate A and the Fleet Satellite Communications. The contributions of the following persons are also gratefully acknowledged: R. A. Marsh and R. J. Mercer for the data on the North American Air Defense Command geosynchronous and low altitude catalogs, K. H. Senechal, R. E. Simberg, and R. P. Gupta for aid in the numerical analyses, and H. K. Karrenberg for many helpful suggestions during the course of this study.

Accession For	
HITTS GDA&I	X
DOTO TAB	Ti
U. bur strated	13
Jackton of them	
Ву -	
District "/	
Aveired	1. Tes
7,441	!'
Dist Spaced	L

CONTENTS

PREFACE		•••••••••••••••••••••••••••••••••••••••	1
I.	INTRO	DUCTION	9
11.	SPACE	OBJECT POPULATION	11
	Α.	Growth Rate	11
	В•	Origin and Distribution	15
	С.	Density of Objects as a Function of Altitude	17
		1. Low Altitude Orbits	17
		2. Geosynchronous Orbits	21
III.	ANALYS	SIS AND RESULTS	33
	Α.	Uniform Density Method	33
	В.	Variable Density Method	37
	C.	Distance of Closest Approach Method	45
		1. General Considerations	45
		2. Miss Distance and Position Uncertainty	45
IV.	SUMMAI	RY AND CONCLUSIONS	59
APPENDI	tx. U	NCLASSIFIED GEOSYNCHRONOUS CATALOG - ALPHABETICAL ORDER	63
REFEREN	NCES		71

FIGURES

1.	Cataloged Objects Population History	13
2.	Computer View of a Sample of Space Objects	14
3.	Known Launches and Payload History	16
4.	SKYNET 1B Geosynchronous Orbit Injection Geometry	18
5.	Cumulative Number of Objects vs. Geocentric Radius in Units of Earth Radius	19
6.	Space Object Density vs. Altitude	20
7.	Fraction of Objects vs. Orbital Farameters	22
8.	Fraction of Objects vs. Inclination	23
9.	Representative Satellite Trajectories Relative to Geopotential Geopotential Stable Points	24
10.	NORAD Unclassified Geosynchronous Catalog	25
11.	Geostationary Communication Satellites	26
12.	Cumulative Number vs. Inclination for Geosynchronous Objects	28
13.	Cumulative Number vs. Eccentricity for Geosynchronous Objects	29
14.	Object Density in Geosynchronous Orbits	30
15.	Right Ascension of the Ascending Node in Geosynchronous Orbits	31
16.	Toroidal Geometry for Geosynchronous Orbits	36
17.	Probability of Collision - Low Earth Orbits (800 to 1500 km) for 1000 Days	38
18.	Probability of Collision - Geosynchronous Orbits for 1000 Days	39
19.	Dwell Fraction vs. Latitude for Circular Orbits	41
20.	Collision Probability for Objects in Circular Geosynchronous Orbits in 1000 Days	43
21.	Collision Probability for Objects in Eccentric Geosynchronous	<i>1</i> . <i>1</i> .

FIGURES (Continued)

22.	Encounter Geometry for Mutually Inclined Circular Orbits	46	
23.	Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 20 \text{ ft}$)	52	
24.	Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 50 \text{ ft}$)	53	
25.	Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 50$ ft) (Continued)	54	
26.	Collision Probability as a Function of Position Uncertainty and Miss Distance (R_S = 100 ft)	55	
27.	Collision Probability as a Function of Position Uncertainty and Miss Distance (R_S = 100 ft) (Continued)	56	
28.	Maximum Collision Probability Per Pass (P_{max}) as Function of Effective Collision Radius to Miss Distance Ratio ($R_S/R_{mi.n}$)	57	
29.	WESTAR-A/OPS 6391 Collision Probability During the April 14 to April 23, 1980 Encounters in Geosynchronous Orbit	58	
30.	Operational Procedures to Minimize Collision Hazard in Space	62	
TABLES			
l.	NORAD Catalog of all Objects for 27 April 1980	12	
2.	Close Encounters Between OPS 6391 and WESTAR-A Geosynchronous Satellites		
3.	Summary of Present and Projected Collision Probabilities for a 1000-Day Mission	60	

I. INTRODUCTION

Since 1957 the continuous use of space has resulted in the buildup of a large number of space objects such as active and spent payloads, rocket bodies, and miscellaneous debris, including numerous explosion fragments. A catalog of an estimated radar-trackable population of some 5000 satellites is maintained by the North American Air Defense Command (NORAD) and other agencies. These objects are a subset of an unknown but larger population of objects including those that are too small to be tracked by radar, but large enough to cause damage to another spacecraft by collision.

In view of this, much interest and concern have been expressed recently by several government agencies and international organizations regarding the orbiting debris problem and the associated collision hazard. Questions on the subject have been posed by the United Nations, NASA and the USAF/SD which have resulted in some assessments of the collision hazard at present and in the near future. Also, work is currently underway to revise the USAF/SD Regulation 550-11 to establish Space Division guidelines and responsibilities for program directors and offices involved in the management of geosynchronous satellites.

The purpose of the present study is to update the existing data on the orbiting population (Ref. 1), and to reexamine the collision hazard issues as currently perceived. Probabilities of collision at low altitudes and in the geosynchronous corridor will be estimated using approximate methods developed in the study. Potential operational and design approaches that can aid in reducing the space debris population growth rates will be outlined. It is hoped that these efforts will contribute towards a better understanding and awareness of the collision hazard so that appropriate action can be taken to maintain a relatively low risk environment for current and future satellite systems.

II. SPACE OBJECT POPULATION

A. GROWTH RATE

On the basis of a worldwide network of sensors, NORAD tracks and maintains a catalog of most of the manmade orbiting objects in space. Although NORAD is the primary source of orbit elements for these objects, other sources are NASA Satellite Situation Reports, RAE tables of earth satellites, TRW Space Log and various military and civilian agencies including those of the United Nations.

The orbiting population of objects consists of operational and decayed payloads, miscellaneous mission-related debris such as rocket bodies, clamps, shrouds, etc., and a multitude of explosion fragments. The latter constitute more than 60% of the trackable population (Ref. 2). For example, the number and type of objects listed in the NORAD catalog for 27 April 1980 are given in Table 1 (Ref. 3). It should be noted, however, that the cataloged population includes only those objects which can be tracked by radar (greater than 0.01 m^2 in area) up to 5000 km in altitude or optically. The currently estimated total population (down to 1 cm² in size) is believed to be on the order of 10,000 to 15,000 objects. In the geosynchronous corridor, the total population down to 1 cm² is probably as much as an order of magnitude greater than that which is cataloged. More detail on the synchronous satellite catalog is provided in the Appendix.

The approximate growth rate of the cataloged population is exhibited in Fig. 1. The observed rate of increase (on the basis of the smoothed data of Fig. 1) is between 9 and 13% per year depending on the time period of interest. An instructive computer illustration of the space objects which could be seen by an observer in a 300 nmi circular orbit for a period of 10 seconds is displayed in Fig. 2 where the apparent size of the object (circle radius) denotes relative distance from the observer.

Table 1. NORAD Catalog of All Objects for 27 April 80

Earth Orbit Payloads			1055	
Earth Orbit Debris			3384	
Deep Space Payloads			61	
Deep Space Debris			52	
	Current C	Objects	4552	4552
Decayed Payloads			1399	
Decayed Debris			5831	
	Decayed (Objects	7230	<u>7230</u>
Total Objects				11782

Synchronous Population - 28 April 80

Satellites

DOD(SCF)/NATO	23	
NASA	17	
COMSAT CORP	13	
LINCOLN LABS, RCA, W.U., ESA	10	
USSR	16	
UK, CAN, JAP, FR, IT, INDONESIA	18	
	97	97
Rocket Bodies and Old Satellites		
With Current Tracking	47	
No Recent Tracking	<u>56</u>	
	103	103
Total Synchronous		200

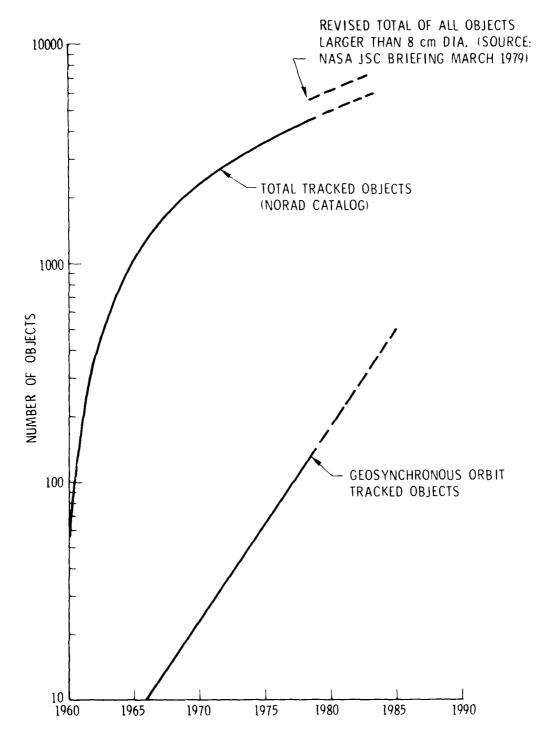


Fig. 1. Cataloged Objects Population History

7 4 - LONGITUDE Ç/ OBSERVER LATITUDE = 34.6^{o} N, LONGITUDE = 239^{o} E, ALTITUDE = 300 nmi AIMPOINT ABOVE LATITUDE = 21.4^{o} N, LONGITUDE = 201.7^{o} E NOTE: APPARENT SIZE INDICATES RELATIVE DISTANCE FROM OBSERVER. FIELD-0F-VIEW IS $30^{\rm o} \times 55^{\rm o}$ q 10.0 deg ABOVE HORIZON MOTION IS FOR 10 sec ζ

Fig. 2. Computer View of a Sample of Space Objects in a 300 nmi Circular Orbit

- LATITUDE

- LONGITUDE

B. ORIGIN AND DISTRIBUTION

The principal sources of the orbiting population of objects to date have been the nearly 2000 launches by the U.S., the USSR, and several other countries. The history of known launches and payloads placed in orbit since 1975 is exhibited in Fig. 3. The principal U.S. launch vehicles are the Delta, Atlas/Agena, Centaur, Titan III and the Scout stages which have accounted for the estimated 33 U.S. launches in 1978. The total of 87 USSR launches in 1978 have resulted in a large number of the COSMOS series of satellites some of which were returned to earth or commanded into a destructive reentry into the atmosphere. By 1 January 1979 there were six launches by France, four by Japan, two by China and one by the U.K. From 1968 the yearly total number of launches varied between 106 and 128 for all countries (Ref. 4).

Payload and rocket body explosions in orbit are another source of space debris. It is known for example, that there have been over 50 such explosions as of 4 October 1978. A number of spacecraft explosions are known to have taken place in orbit related to antisatellite tests (ASATs) and/or attempts to inject payloads into geosynchronous orbit. A recent example of the latter was the apparent loss of the RCA Satcom 3 and the Japanese ECS 2 satellites which seem to have experienced anomalous burns of the apogee injection motors that resulted in a loss of telemetry before the completion of the insertion burn. Another example of such a malfunction is the SKYNET 1B launch on 19 August 1970 on a Thor-Delta vehicle which placed a communication payload into a 137 \times 20099 nmi transfer orbit. On 22 August 1970, the apogee burn motor (ABM) was fired which should have placed the satellite into a nearly synchronous equatorial orbit. Approximately 14 seconds after the initiation of the apogee burn, all tracking and telemetry data were abruptly lost. Subsequent analyses of the data (Refs. 5 and 6) indicated that the ABM performed anomalously resulting in "regressive burning." An estimated ΔV = 4000 fps out of the required $\Delta V = 5814$ fps appears to have been added at apogee. The resultant orbit for the assumed explosion fragments was estimated as approximately 5192×20088 nmi inclined 5.30° to the equator. The velocity at apogee relative to the geosynchronous velocity is about 1700 fps which may represent

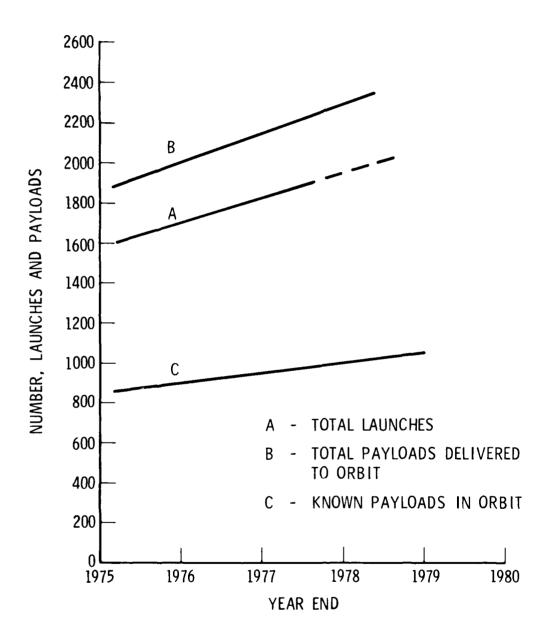


Fig. 3. Known Launches and Payload History

a significant collision hazard to any geosynchronous object in the vicinity of this orbit. The number and distribution of the explosion fragments, if an explosion in fact occurred, cannot be easily determined. However, the high perigee altitude of the final orbit precludes any significant reduction of the debris because of atmospheric or other perturbative effects. Figure 4 exhibits the orbit geometry for this event.

Other potential sources of space debris are collisions between debris objects and spacecraft or other debris objects. It is considered likely, for example, that the solar panel malfunction on the geosynchronous GEOS-2 spacecraft is a result of a collision with a small object (Ref. 7). Also, the breakup of PAGEOS balloon in July, 1975 after 9 years in orbit appears to have resulted from a similar cause. The low orbit COSMOS 954 was similarly reported to have suffered a sudden depressurization thought to be caused by a collision with debris (Ref. 2). Air Force satellite programs in synchronous orbit have reported, on two separate occasions, sudden small changes in spacecraft angular momentum. These unexplained changes may have been caused by impacts with natural or manmade objects.

The possibility of a self-perpetuating debris belt caused by intercollisions between objects in space also exists. This process would parallel certain theories concerning the growth of the asteroid belt. The debris flux in such a belt could exceed the natural meteoroid flux in the not too distant future if present trends continue (Ref. 8).

C. DENSITY OF OBJECTS AS A FUNCTION OF ALTITUDE

LOW ALTITUDE ORBITS

A total of 4174 space objects from the NORAD Catalog for April 28, 1980 have been examined numerically to determine the distribution with altitude. The results are presented in Figs. 5 and 6. It can be seen that the majority of satellites are within 1.5 earth radii (E.R.) with a small increase to synchronous altitude where an increment occurs at 6.62 E.R. Figure 6 indicates that the greatest density of objects occurs between 500 and 1500 km altitude. Distributions by semimajor axis A (normalized with respect to earth

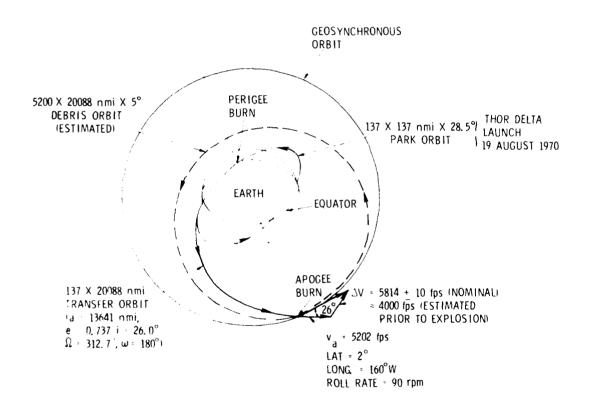


Fig. 4. SKYNET 1B Geosynchronous Orbit Injection Geometry

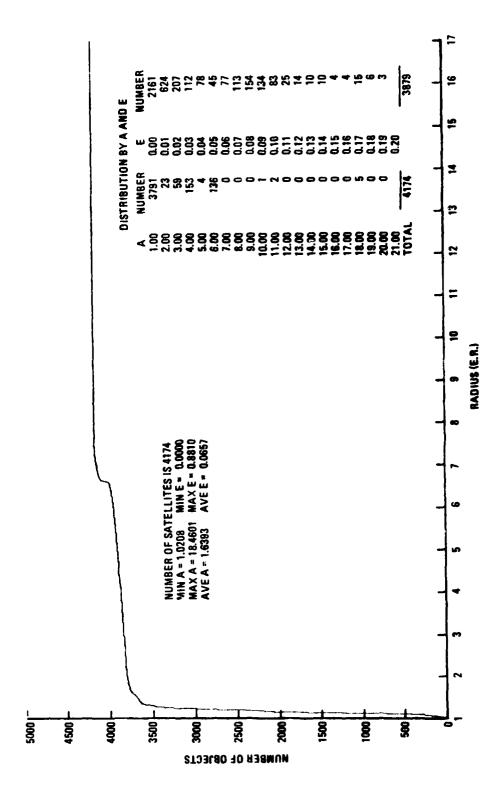


Fig. 5. Cumulative Number of Objects vs. Geocentric Radius in Units of Earth Radius

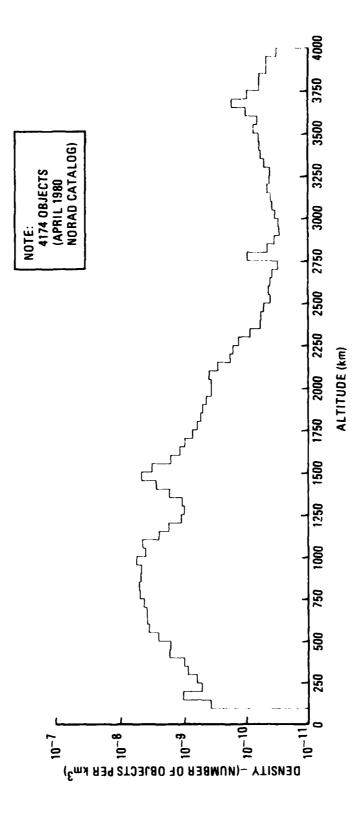


Fig. 6. Space Object Density vs. Altitude

radius), eccentricity E, ascending node (NODE) and the argument of perigee (OMEGA) are given in Fig. 7. It can be seen, for example, that there are 3791 objects with A greater than 1 but less than 2, and E greater than zero but less than 0.01. The distribution with respect to orbital inclination (INC) is given in Fig. 8.

GEOSYNCHRONOUS ORBITS

The geosynchronous region of space is a unique corridor where most of the present and future communication satellites are or will be located. The principal reason for this is that these satellites appear to be either stationary with respect to the earth or are slowly drifting relative to some initial longitude. The satellites are stationary when they are in circular orbits in the equatorial plane with periods of revolution equal to the rotational period of the earth. Secular or continuous drifting occurs due to slight differences in the orbital periods of revolution. Periodic drifting may also occur because of orbital eccentricity or inclination effects as well as perturbing effects of the geopotential field. For example, the ${\rm J}_{2,2}$ tesseral harmonic causes an oscillatory (librational) motion about the geopotential stable points at 75.3°E and 255.3°E longitudes with long periods (on the order of years) of libration (see Fig. 9). A large number of objects librating about these points may pose an additional hazard to the active pavloads placed at or near these locations. The longitudinal location for 154 geosynchronous objects listed in the appendix is illustrated in Fig. 10. NORAD publishes the orbital elements on these objects on the basis of data obtained from NORAD, the Satellite Control Facility (SCF), NASA and Intelsat tracking networks. The data on the orbital elements is provided by NORAD in the form of punched cards received via TWX on a daily basis. Updates for any given object are provided typically within a one or two week period.

Figure 11 presents the longitudinal location of the current and planned communication satellites in geosynchronous orbit (Refs. 9 to 11). Many of these comsats are station-kept within small longitudinal and latitude bands while others drift as the result of natural forces. A few have also been removed from geosynchronous orbit after the completion of their mission (e.g.,

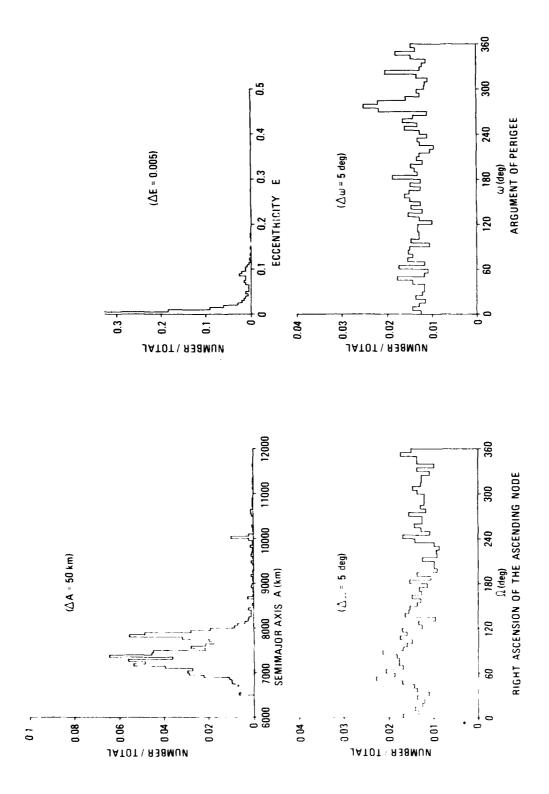


Fig. 7. Fraction of Objects vs. Orbital Parameters

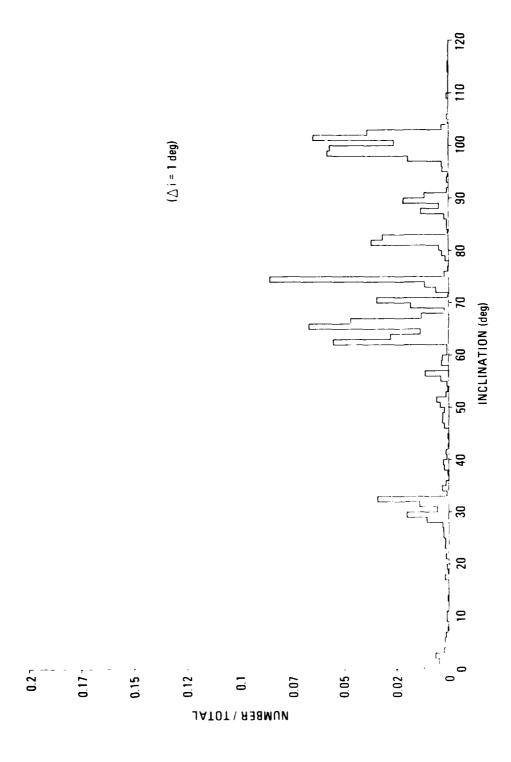
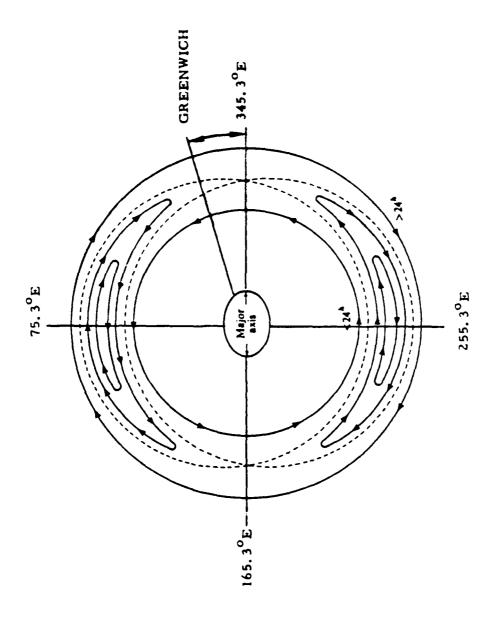


Fig. 8. Fraction of Objects vs. Inclination



Representative Satellite Trajectories Relative to Geopotential Stable Points Fig. 9.

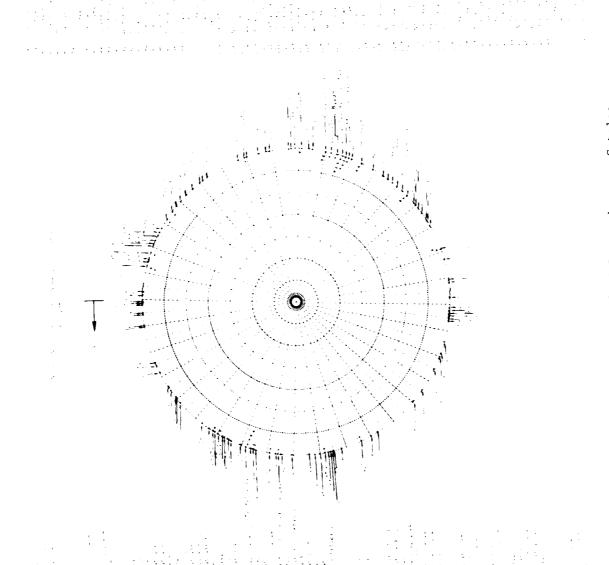


Fig. 10, YORAD Unclassified Geosynchronous Catalog

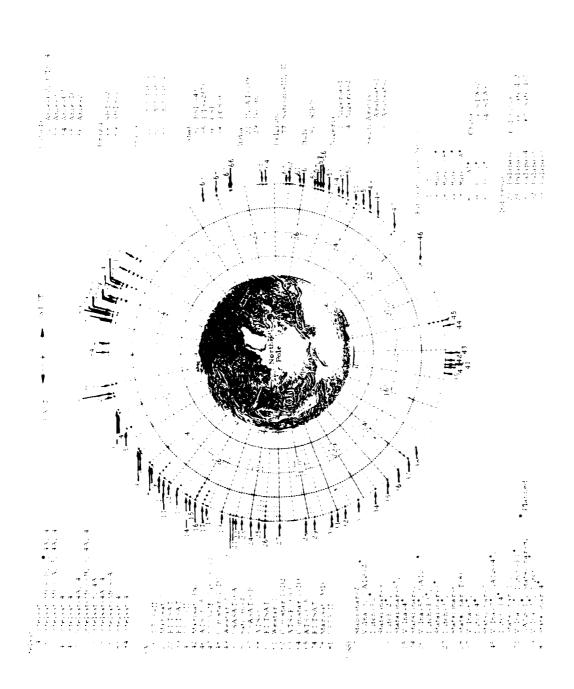


Fig. 11. Geostationary Communication Satellites

Intelsat 3, F-2, F-4 and F-6 to altitudes of 400 to 3700 km above synchronous altitude)(Ref. 12). The ATS-F has been removed from the synchronous altitude orbit to an altitude 250 nmi below synchronous. The distribution of a sample of 134 geosynchronous objects with inclination and eccentricity is given in Figs. 12 and 13, respectively. These results indicate that typical geosynchronous orbits are nearly circular with low inclinations relative to the equator. A few satellites (e.g. ELECTRON, and their rocket bodies have inclinations above 60° (see Appendix). Object density, illustrated in Fig. 14, decreases as the altitude from the geosynchronous orbit and latitude increase. The results in Fig. 14 were obtained numerically by taking "snapshots" of objects once each hour over a period of twenty-four hours within a torus of latitude band $\Delta \varphi$ and altitude band Δh centered about the geosynchronous altitude. The total number of observations was then summed and divided by 24 to get an average number of sightings in each band. The average density is the number of sightings divided by the volume of the torus. The nonuniformity of the inclinations of the objects orbits within each latitude band is illustrated in Fig. 14 by the average inclination $\overline{1}$ as noted.

The ascending node distribution for 154 geosynchronous objects presented in Fig. 15 indicates that the orbital planes are not distributed uniformly, but tend to be concentrated in certain regions of space. The impact of this is to increase the probability of collision for some satellites above the representative figures calculated for the synchronous class of satellites.

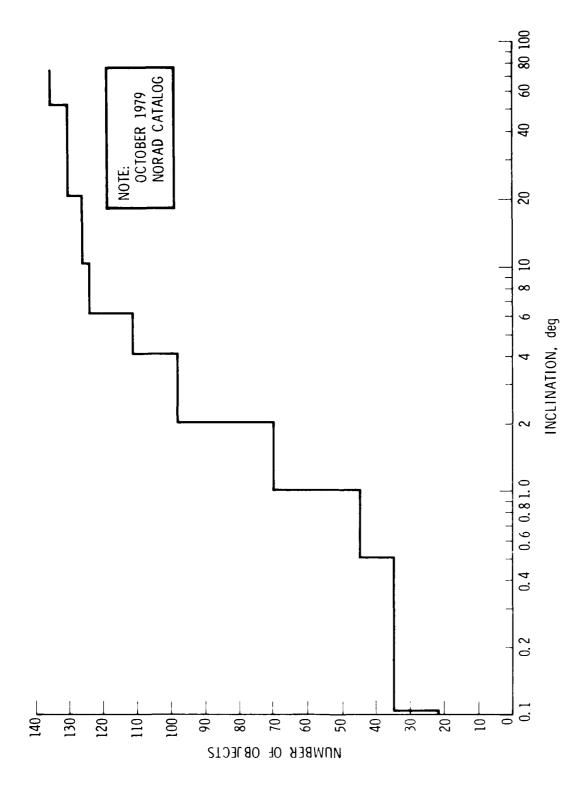


Fig. 12. Cumulative Number vs. Inclination for Geosmorphronous Objects

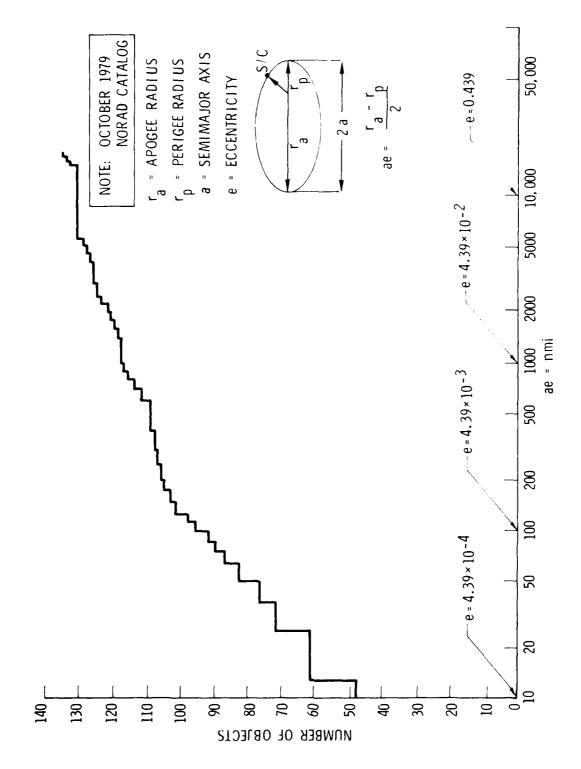


Fig. 13. Cumulative Number vs. Eccentricity for Geosynchronous Objects

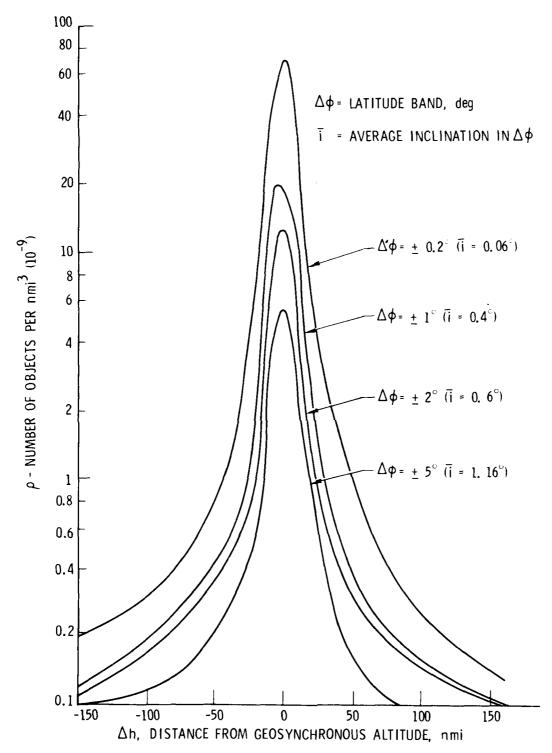


Fig. 14. Object Density in Geosynchronous Orbits

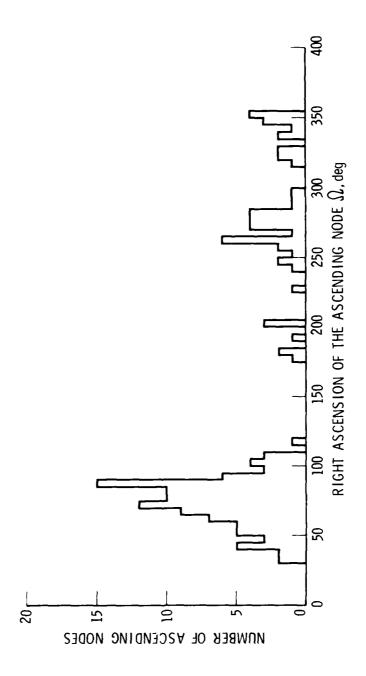


Fig. 15. Right Ascension of the Ascending Node in Geosynchronous Orbits

III. ANALYSIS AND RESULTS

A. UNIFORM DENSITY METHOD

The probability that any two satellites will collide is generally a function of their orbital parameters, size and time. The collision cannot take place until the orbits intersect or approach each other to within the dimensions of the satellites of interest. This may occur even for initially nonintersecting orbits because of the effects of the earth's oblateness, air drag and solar-lunar perturbations which alter the orbital parameters in time.

An approximate, yet reasonably accurate method for estimating the probability of collision between a target satellite of projected area A_c and a debris object is based on the assumption that the space object density ρ is uniform (constant) and that the target area A_c sweeps out a volume $A_c\Delta t$ in time Δt . It can be shown (Ref. 1) that the fractional number of objects encountered in the volume A_c Δt is then equal to the probability of collision if it is much less than unity. Thus,

$$p(co1) = \rho v_r A_c \Delta t \qquad (1)$$

where v_r is the average relative velocity between the satellite and the set of objects considered. For example, $v_r \approx 7$ km/sec for circular orbits up to 2000 km in altitude (Ref. 8).

The relative velocity $v_{\bf r}$ at the trace intersection between any two satellites in orbit planes with a mutual inclination of Δi can be expressed as (Ref. 1):

$$v_{r} = \sqrt{\frac{\mu}{r_{x}}} \left\{ 4 - \left(\frac{1}{A_{1}} + \frac{1}{A_{2}}\right) - 2\sqrt{\left[2 - \frac{1}{A_{1}} - A_{1}(1 - e_{1}^{2})\right]} \left[2 - \frac{1}{A_{2}} - A_{2}(1 - e_{2}^{2})\right] - 2\sqrt{A_{1}(1 - e_{1}^{2})A_{2}(1 - e_{2}^{2})} \cos \Delta i \right\}^{1/2}$$
(2)

where r_x is the radius at trace intersection and $A_1 = a_1/r_x$. $A_2 = a_2/r_x$. Here a_1 , a_2 , e_1 , e_2 , μ are the semimajor axes, eccentricities and the gravitational constant, respectively.

For the case of a circular and an eccentric orbit (e.g. e_2 = 0, r_x = a_2) Eq. (2) becomes

$$v_r = \sqrt{\frac{\mu}{r_x}} \left[3 - \frac{1}{A_1} - 2\sqrt{A_1(1 - e_1^2)} \cos \Delta i \right]^{1/2}$$
 (3)

For circular orbits of equal period (i.e., $a_1 = a_2$, $e_1 = e_2 = 0$), Eq. (2) reduces to

$$v_{rc} = 2v_{c} \sin \frac{\Delta i}{2}$$

$$\approx v_{c} \sin \Delta i \qquad \text{for } \Delta i \text{ small.}$$
(4)

which is equal approximately to the cross-track (normal) component of relative velocity at encounter with v_c = circular orbital velocity. An average relative velocity \overline{v}_{rc} for this case can be defined as an average value of v_{rc} over a mean anomaly range $-\pi/2 \le M \le \pi/2$, i.e.

$$\frac{1}{v_{rc}} \approx \frac{v_{c}}{\pi} \sin \Delta i \int_{-\pi/2}^{\pi/2} \cos M dM$$

$$\approx \frac{2v_{c}}{\pi} \sin \Delta i \qquad (5)$$

An approach which can be applied to the case of a spacecraft in a circular, inclined orbit is based on the determination of object density in a spheroidal torus containing the spacecraft orbit (Fig. 16). This approach is similar to that used in the theory of interplanetary encounters (Ref. 13). The spheroidal torus is defined by the relative orbit plane inclination Δi and the spacecraft orbit radius R. The probability of the object colliding with the spacecraft in a time interval Δt is

$$p(col/\Delta t) = 1 - e^{-\lambda \Delta t}$$

$$\approx \lambda \Delta t \qquad \text{for } \lambda \Delta t \iff 1$$
(6)

where

$$\lambda = \rho v_r A_c$$

$$= \frac{S^2 v_r}{2 T |u_r| \sin \Delta i}$$

= impact rate per unit time

T = object orbit period of revolution

 $S = R_s/R$

 R_s = spacecraft radius

 v_r = relative velocity at encounter

 $|u_r|$ = absolute value of the radial component of v_r

For a circular target orbit and an eccentric object orbit (Refs. 1, 13)

$$\frac{v_{r}}{|u_{r}|} = \left[\frac{3 - \frac{1}{A_{1}} - 2\sqrt{A_{1}(1 - e_{1}^{2})\cos \Delta t}}{2 - \frac{1}{A_{1}} - A_{1}(1 - e_{1}^{2})} \right]^{1/2}$$
(7)

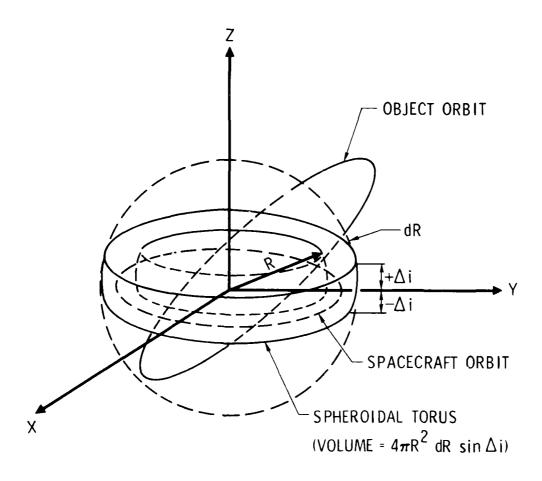


Fig. 16. Toroidal Geometry for Geosynchronous Orbits

where $A_1 = a_1/a_2$. Also, a_1 , a_2 , are the semimajor axes of the object and target orbits, and e_1 is the eccentricity of the object orbit.

The probability of the target spacecraft colliding with N objects in the torus (defined by an average inclination Δi_{av} of the objects' orbits relative to that of the target orbit) is equal to N times that given by Eq. (6). For example, if a set of 62 geosynchronous objects is considered with an average eccentricity $e_1 = 0.00113$ and orbit plane inclination $\Delta i = 1.07^{\circ}$ relative to the equatorial plane, then the velocity relative to a spacecraft in a circular geosynchronous equatorial orbit at encounter is from Eq. (3) $v_r = 57.5$ m/sec where $a_1 = 1$ and $r_x = \text{geosynchronous radius}$. Consequently, a 1000 day probability of collision for a spacecraft of 50 m radius is by Eq. (6) equal to 3.88×10^{-5} .

Using Eq. (1), $v_r = 7 \text{ km/sec}$, which is the average relative encounter velocity between satellites below 2000 km (Ref. 8) and the spatial density ρ in Fig. 7, the probability of collision for circular low altitude missions of 1000 days is given in Fig. 17 for the trackable population of 4174 objects and an estimated population of 8400 objects of smaller cross-section. The results show that the 1000 day probability of collision for a very large spacecraft (~ 50 m radius) is on the order of 4 to 8% and for a 20 m radius satellite it is on the order of 0.6 to 1.2%. However, for smaller spacecraft (~ 3 m radius), current probability of collision is an order of magnitude lower. The results for a sample of 62 to 620 objects in the geosynchronous corridor are given in Fig. 18 where an average relative velocity of only 36.5 m/sec was assumed. This velocity is the north-south (inclination) component of the average relative velocity. Average collision probabilities in the geosynchronous orbit are thus seen to be about three orders of magnitude lower than those at low altitudes. However, the effects of density variation with altitude, inclination (latitude) or longitude are not included in these results.

B. VARIABLE DENSITY METHOD

Figure 14 shows that the observed object density $\rho(h,\phi)$ is a function of height h above or below geosynchronous orbit and latitude ϕ . Since a satellite whose orbit plane is inclined to the equatorial plane spends only a

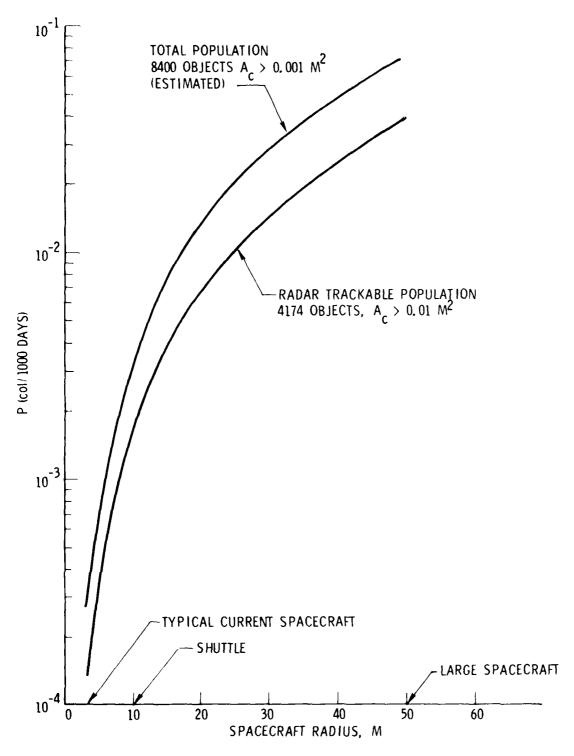


Fig. 17. Probability of Collision - Low Earth Orbits (800 to 1500 km) for 1000 Days

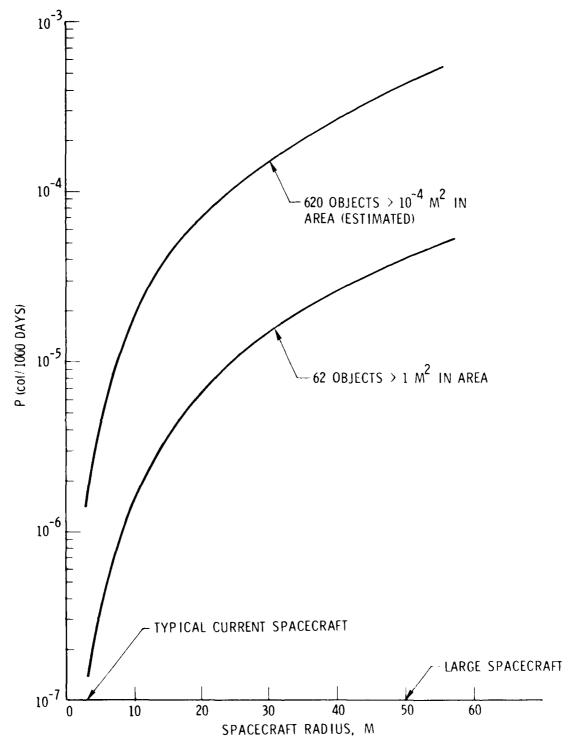


Fig. 18. Probability of Collision - Geosynchronous Orbits for 1000 Days

fraction of its period of revolution T near the equator where the density is greatest, the probability of collision is similarly affected. In general, the probability of collision in any altitude band $\Delta h = h_2 - h_1$, per one revolution of the satellite, is

$$p(col/rev) = A_c \bar{v}_r T \int_{h_1}^{h_2} \int_{-\phi}^{\phi} \rho(h,\phi) f(\phi) d\phi dh$$
 (8)

where $\rho(h,\phi)$ is the object density function, $f(\phi)$ is a weighting function which can be derived from the time fraction spent by a satellite in a latitude band $\Delta \phi$ as shown in Fig. 19 and \overline{v}_r is the average velocity of the target satellite relative to the objects of interest. For the case of a target satellite with inclination i and a set of geosynchronous objects with an average inclination \overline{i} and average right ascension of the ascending node $\overline{\Omega}$, the angle Δi between the target orbit plane and that of the "average plane" of the objects is

$$\Delta i = \cos^{-1}(\cos i \cos \overline{i} + \sin i \sin \overline{i} \cos \Delta \Omega) \tag{9}$$

where

$$\Delta\Omega = |\overline{\Omega} - \Omega|$$

 Ω = right ascension of the ascending node of the target satellite.

Since Δi will in general vary in time because of external perturbations, the average value of Δi is

$$\Delta i_{av} = \frac{\Delta i_{max} + \Delta i_{min}}{2} \tag{10}$$

i - SATELLITE ORBIT INCLINATION

T = ORBIT PERIOD t = TIME IN LATITUDE BAND ± φ

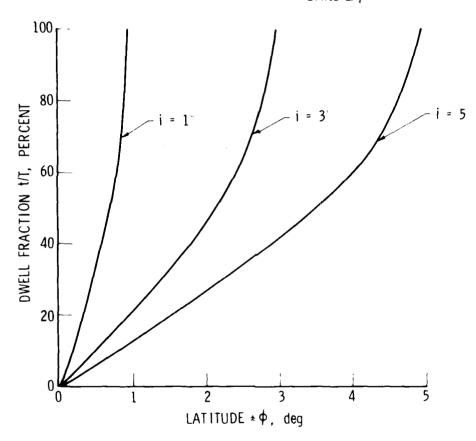


Fig. 19. Dwell Fraction vs. Latitude for Circular Orbits

where

$$\Delta i_{\text{max}} = i + \overline{i} \quad \text{when} \quad \Delta \Omega = \pi$$
 (11)

and

$$\Delta i_{\min} = |i - \overline{i}| \text{ when } \Delta \Omega = 0$$
 (12)

Therefore,

$$\Delta i_{av} = \begin{cases} i & \text{for } i > \overline{i} \\ \overline{i} & \text{for } i < \overline{i} \end{cases}$$
 (13)

The average relative velocity $\mathbf{v_r}$ for the case of nearly circular geosynchronous orbits can be approximated by Eq. (5) because the normal (out-of plane) or north-south component predominates.

Equation (8) has been evaluated approximately for circular orbits with i = 1°, 3° and 5° and object densities of Fig. 14 for different altitude bands above and below geosynchronous orbit. The results, given in Fig. 20, are valid for a typical small satellite (effective collision radius R_S = 20 ft (6.1 m)) and a mission duration of 1000 days. The results for other values of R_S are proportional to the square of the effective radius of collision and time. The collision probability for a target satellite in an elliptic geosynchronous orbit can be obtained by averaging the results of Fig. 18 as shown in Fig. 21.

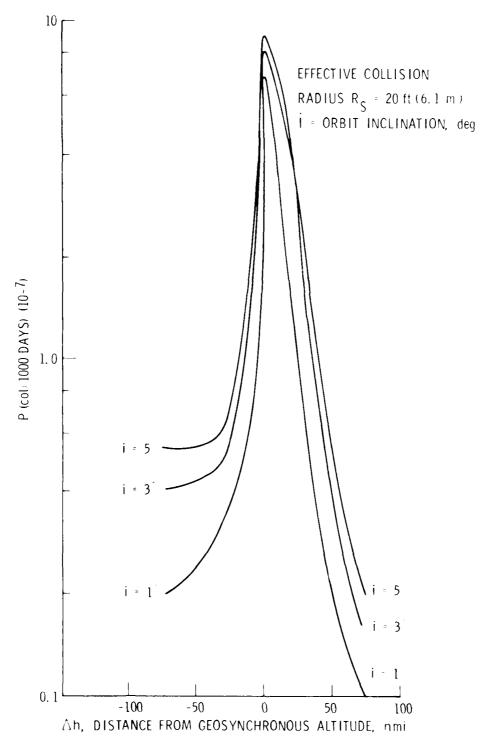


Fig. 20. Collision Probability for Objects in Circular Geosynchronous Orbits in 1000 Days

EFFECTIVE COLLISION
RADIUS R = 20 ft (6.1 m)
i = ORBIT SINCLINATION

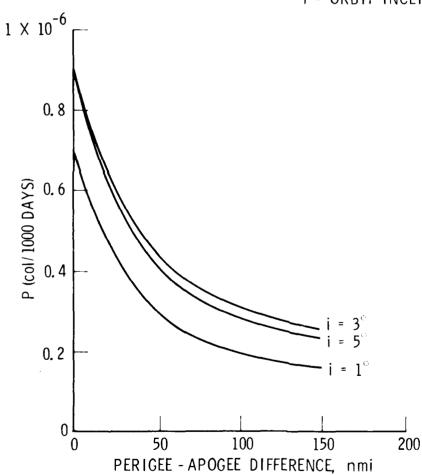


Fig. 21. Collision Probability for Objects in Eccentric Geosynchronous Orbits in 1000 Days

C. DISTANCE OF CLOSEST APPROACH METHOD

1. GENERAL CONSIDERATIONS

A collision between two objects can take place at or near a trace intersection point if the distance of closest approach $R_{\mbox{\footnotesize{MIN}}}$ is equal to or is less than the effective collision radius $\mathbf{R}_{S^{\bullet}}$. For satellites in circular orbits with a mutual orbit plane inclination $\alpha,\ R_{\mbox{\scriptsize MIN}}$ occurs twice per revolution in the vicinity of the intersection between the orbit planes (nodal axis) as illustrated in Fig. 22. For each instant when satellite 1 is at or near the nodal axis, satellite 2 is at a position 4, 3, 2 or 1 corresponding to an R_{MIN} for that pass. The angular increment $\Delta u = n_1 | (T_2 - T_1) |$ where n_1 is the mean motion of satellite 1 and T_1 , T_2 are the periods of revolution for satellites 1 and 2, respectively. The angular change Δu per revolution is typically a fraction of a degree for geosynchronous satellites. For example, $R_{
m MIN}$ for north and south bound passes of OPS 6391 (SDC object No. 10669) and WESTAR-A (SCD object No. 7250), is given in Table 2 for several days in April of 1980 as was determined approximately via the numerical simulation described in Ref. 14. Different R_{MTN} values can thus be seen to have occurred before and after the lowest value $R_{MIN} = 5.28$ nmi of 21 April 1980. The probability of collision between the objects for each pass is, in general, a function of R_{MTN} , the tracking uncertainty σ in the location of each object and the effective collision radius R_{ς} . The latter can be defined as one half the sum of the maximum dimensions for both objects. For σ = 0 and $R_{\rm MIN} > R_{\rm S}$ no collision can take place. For $\sigma \neq 0$, there is a nonzero probability of collision.

MISS DISTANCE AND POSITION UNCERTAINTY

An approximate collision probability method, described briefly in (Ref. 15), considers the effects of the uncertainties associated with the three dimensions (coordinates) of the miss distance $R_{\rm MIN}$. It is assumed that the uncertainties are Gaussian (normal) with zero biases and equal variance and that they are uncorrelated. The assumption is applied to the position data of each of the tracked satellites. In view of this and the fact that the orientation of the coordinate system containing the distance of closest

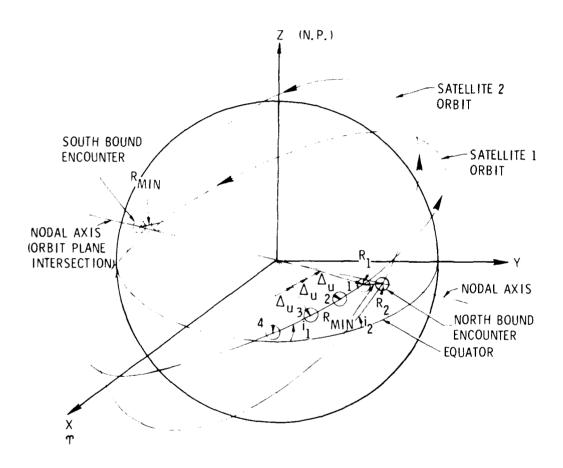


Fig. 22. Encounter Geometry for Mutually Inclined Circular Orbits

Close Encounters Between OPS 6391 and WESTAR-A Geosynchronous Satellites Table 2.

					106	10669 (OPS 6391	1)	72	7250 (Westar-A	·A)
RMIN- (1	R _{MIN} (Relative Minimum) (nmi/km)		Ď	Date	lat (deg)	long. (deg)	alt (nmi)	lat (deg)	long. (deg)	alt (nmi)
26.55/	49.17	12.	Apr	9:42:58.1	.0317	-99.2406	19329.05	.0337	-99.1751	19323.83
14.31/	26.50	12.	Apr :	21:41: 8.0	0341	-99.1765	19316.98	0336	-99.1437	19322.82
23.63/	43.77	13.	Apr	9:38:58.3	.0316	-99.2429	19329.07	.0334	-99.1850	19323.84
11.68/	21.63	13.	Apr ;	21:37: 5.7	0332	-99.1791	19316.98	0328	-99.1537	19322.83
20.76/	38.45	14.	Apr	9:35: .5	.0307	-99.2455	19329.09	.0322	-99.1950	19323.86
9.26/	17.16	14.	Apr ;	21:33: 5.2	0315	-99.1817	19316.98	0312	-99.1636	19322.84
17.94/	33.22	15.	Apr	9:31: 4.1	.0290	-99.2479	19329.10	.0303	-99.2047	19323.87
7.25/	13.43	15.	Apr ;	21:29: 5.7	0295	-99.1839	19316.98	0292	-99.1732	19322.84
15.16/	28.08	16.	Apr	9:27: 8.2	.0272	-99.2499	19329.12	.0282	-99.2141	19323.88
6.02/	11.15	16.	Apr ;	21:25: 6.3	0274	-99.1858	19316.99	0272	-99.1824	19322.85
12.44/	23.05	17.	Apr	9:23:12.2	.0256	-99.2515	19329.13	.0264	-99.2231	19323.90
/80•9	11.25	17.	Apr :	21:21: 6.6	0258	-99.1873	19316.99	0257	-99.1913	19322.86
/48.6	18.22	18.	Apr	9:19:15.8	.0247	-99.2529	19329.14	.0253	-99.2319	19323.91
7.41/	13.73	18.	Apr ?	21:17: 6.6	0249	-99.1888	19317.00	0250	-99.2002	19322.87
7.48/	13.86	19.	Apr	9:15:19.1	.0246	-99.2544	19329.16	.0250	-99.2410	19323.92
9.53/	17.65	19.	Apr ?	21:13: 6.2	0251	-99.1907	19317.01	0252	-99.2096	19322.88
5.73/	10.62	20.	Apr	9:11:22.2	.0255	-99.2566	19329.17	.0256	-99.2508	19323.93
12.04/	22.29	20.	Apr (21: 9: 5.6	.0260	-99.1934	19317.02	0262	-99.2198	19322.89
5.28/	9.78	21.	Apr	9: 7:24.9	.0270	-99.2598	19329.18	.0269	-99.2615	19323.94
14.74/	27.31	21.	Apr ;	21: 5: 4.5	0274	-99.1971	19317.03	0277	-99.2312	19322.90
6.43/	11.90	22.	Apr	9: 3:27.2	.0288	-99.2642	19329.19	.0285	-99.2735	19323.96
17.56/	32.52	22.	Apr 2	21: 1: 2.9	0290	-99.2021	19317.03	0294	-99.2438	19322.90

approach at encounter is arbitrary, a plane xy which is normal to the relative velocity vector and which contains the vector of closest approach R_{MIN} can always be found. The bi-variate normal density function for this case can be expressed as (Ref. 16)

$$f(x,y) = \begin{pmatrix} -\frac{x^2}{2\sigma^2} \\ \frac{e}{\sigma\sqrt{2\pi}} \end{pmatrix} \begin{pmatrix} -\frac{y^2}{2\sigma^2} \\ \frac{e}{\sigma\sqrt{2\pi}} \end{pmatrix}$$
$$= \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}\left(\frac{R_{MIN}}{\sigma}\right)^2}$$
(14)

where

$$R_{MIN} = (X_{min}^2 + Y_{min}^2)^{1/2}$$

Since a collision can occur only if

$$Y_{\min} - R_{s} \leq y \leq R_{s} + Y_{\min}$$

$$X_{\min} - R_{s} \leq x \leq R_{s} + X_{\min}$$
(15)

where X_{min} , Y_{min} are the coordinates of R_{MIN} and where R_s is the effective collision radius for both satellites, the probability of collision is

$$p(col) = \int_{(X_{\min} - R_s)}^{(X_{\min} + R_s)} \int_{(Y_{\min} - R_s)}^{(Y_{\min} + R_s)} f(x,y) dy dx$$
 (16)

$$\approx \frac{2}{\pi} \left(\frac{R_s}{\sigma}\right)^2 e^{-\frac{1}{2} \left(\frac{R_{MIN}}{\sigma}\right)^2} \tag{17}$$

for $R_s \ll R_{MIN}$.

Equation (17) represents the probability of collision between two satellites per encounter when the distance of closest approach $R_{\mbox{\footnotesize{MIN}}},$ the 1 σ tracking uncertainty in the position of each satellite, and the effective radius of collision $R_{\mbox{\footnotesize{S}}}$ are given.

A similar result can be obtained by orienting the x, y coordinates along R_{MIN}^{\bullet} and normal to it respectively. The probability of collision is then

$$p(col) = p_{x}p_{y}$$
 (18)

where

$$p_{\mathbf{x}} = p[\mathbf{A} \le \mathbf{x} \le \mathbf{B}]$$

$$= p\left[\frac{\mathbf{A}}{\sigma} \le \mathbf{x}' \le \frac{\mathbf{B}}{\sigma}\right]$$

$$= \frac{1}{\sqrt{2\pi\sigma}} \int_{\mathbf{A}/\sigma}^{\mathbf{B}/\sigma} e^{-\frac{\mu^2}{2}} d\mathbf{u}$$

= probability of collision along x

$$p_{y} = p[-R_{s} \le y \le R_{s}]$$

$$= p[-\frac{R_{s}}{\sigma} \le y \le \frac{R_{s}}{\sigma}]$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-R_{s}/\sigma}^{R_{s}/\sigma} e^{-\frac{u^{2}}{2}} du$$

= probability of collision in a direction normal to $ar{R_{ ext{MIN}}}$

Here

$$A = R_{MIN} - R_{s}$$

$$B = R_{MIN} + R_{s}$$

and A/o, B/o and $R_{\mbox{\scriptsize g}}/\sigma$ are standardized Gaussian variables.

Assuming now that the probability of collision at j-th closest approach is $p_{\hat{j}}(\text{col})$ the probability of miss is

$$p_{j}(miss) = 1 - p_{j}(col)$$
 (19)

The probability of missing at all n closest approaches is

$$p(miss/n) = \prod_{j=1}^{n} [1 - p_{j}(col)]$$
 (20)

if all probabilities are independent. Therefore, the probability that there will be at least one collision during this period of encounter is

$$p(col) = 1 - p(miss/n)$$

$$\sum_{j=1}^{n} p_{j}(col)$$
(21)

when all $p_{i}(col) \ll 1$.

A plot of Eq. (17) for $R_{\rm S}$ = 20, 50 and 100 ft is given in Figs. 22 to 26 as a function of σ with $R_{\rm MIN}$ as a parameter. It can be seen that a maximum probability of collision occurs when

$$\sigma = \frac{R_{MIN}}{\sqrt{2}} \tag{22}$$

which is plotted as a function of $R_{\rm S}/R_{\rm MIN}$ in Fig. 28. The use of Eqs. (17) and (21) is illustrated in Fig. 29 for the case of the close encounters between the WESTAR-A and the OPS 6391 satellites given in Table 2. The collision probabilities for several passes are plotted as a function of σ . The maximum probability of collision for 17 passes is given as the sum of the maximum probabilities for each pass. Assuming no correlation between passes, the maximum (upper-bound) probability of collision for these encounters is 2.0×10^{-6} for the 8-day time period.

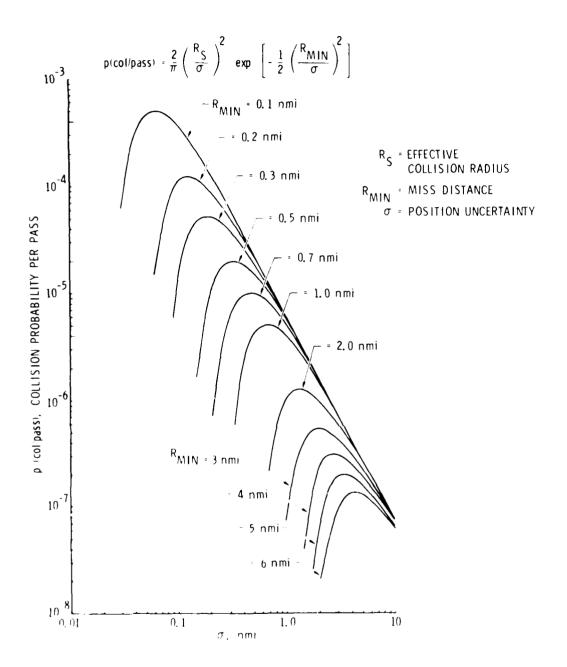


Fig. 23. Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 20$ ft)

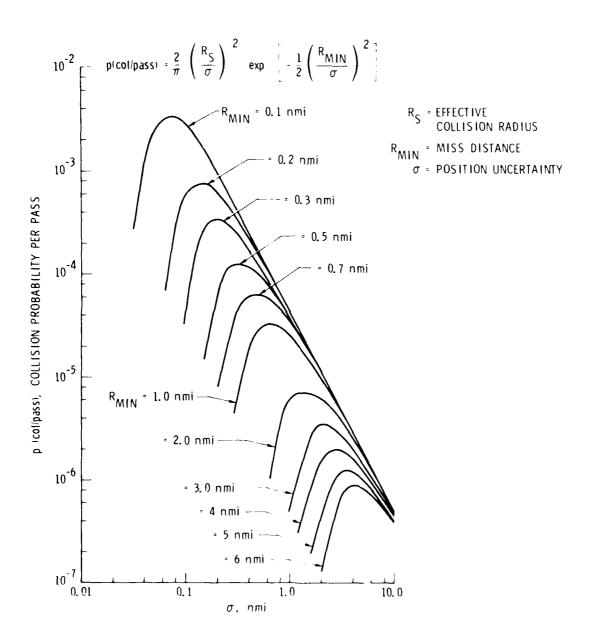
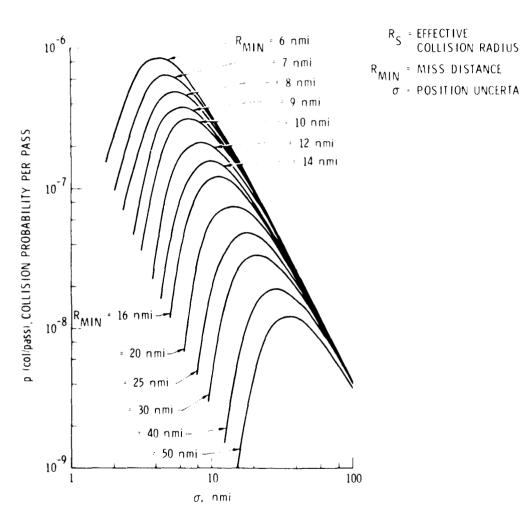


Fig. 24. Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 50$ ft)

$$p(cot/pas \, s) = \frac{2}{\pi} \left(\frac{R_S}{\sigma} \right)^{-2} = exp \left[-\frac{1}{2} \left(\frac{R_M \, IN}{\sigma} \right)^{-2} \right]$$

 σ = POSITION UNCERTAINTY



Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 50$ ft), Continued

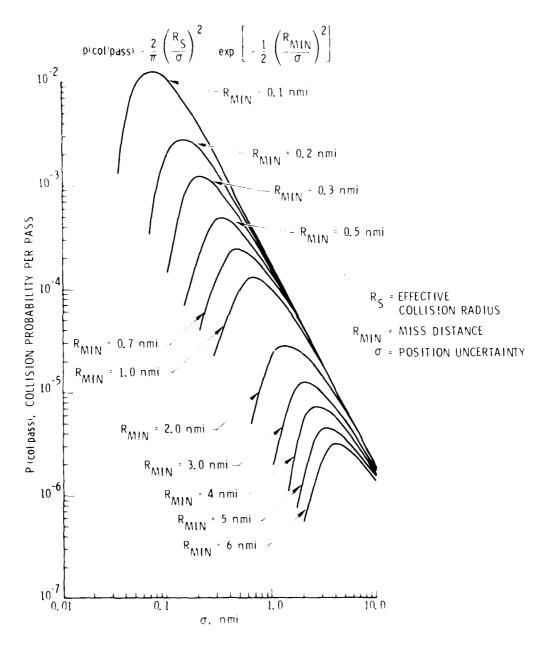


Fig. 26. Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 100 \text{ ft}$)

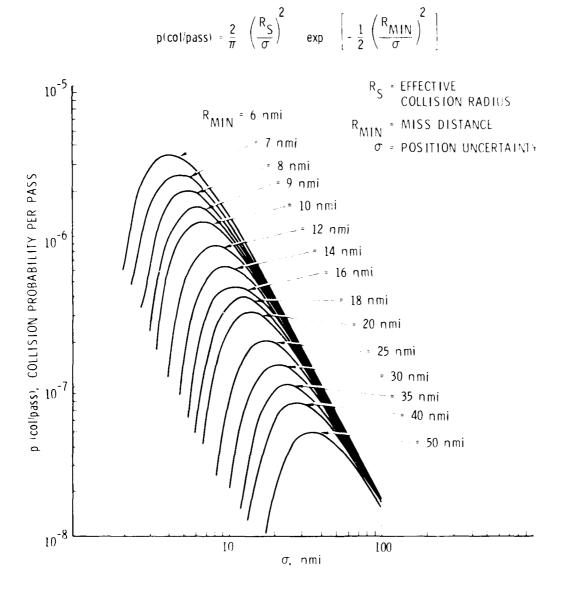


Fig. 27. Collision Probability as a Function of Position Uncertainty and Miss Distance ($R_S = 100$ ft), Continued

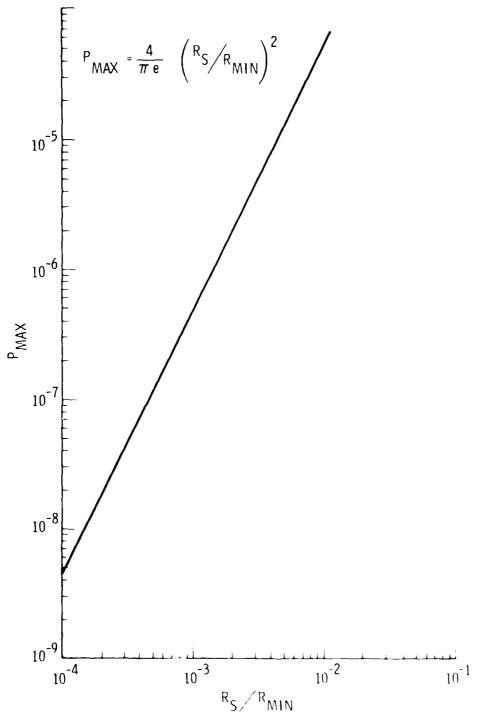


Fig. 28. Maximum Collision Probability Per Pass (P) as Function of Effective Collision Radius to $^{\rm mox}_{\rm obs}$ Distance Ratio (R_S/R_{min})

37

EFFECTIVE COLLISION RADIUS R_S = 30 ft, $p(col)_{max} = \frac{17}{\sum_{i=1}^{17}} P_{imax} = 2.0 \times 10^{-6}$

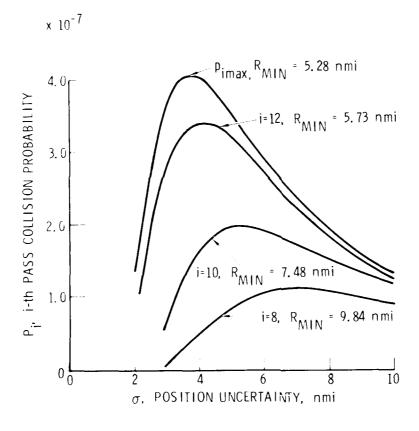


Fig. 29. WESTAR-A/OPS 6391 Collision Probability During the April 14 to April 23, 1980 Encounters in Geosynchronous Orbit

IV. SUMMARY AND CONCLUSIONS

A summary of the present and projected probabilities of collision for a ten meter radius spacecraft in low earth orbit (LEO) and the geosynchronous corridor (GEO) is given in Table 3. It can be seen that the probability of collision in low earth orbits is currently on the order of 0.15 to 0.3% for a 1000-day mission. Another way of interpreting the probability of 0.3% is that one of 90 such spacecraft would be expected to experience a collision in 10 years. Probabilities for the 1985 to 1995 time frame are on the order of 2.5 to 7.3 times current values on the basis of an assumed debris growth rate of 13% per year. On the other hand, the results for the geosynchronous orbit are some three orders of magnitude lower in value mainly because of lower relative velocities at encounter. These results, however, do not include the effects of ascending node concentrations at certain longitudes. The calculated probabilities of collision at low altitude are also likely to be too low due to the noninclusion of many smaller particles believed to be a result of satellite explosions and ASAT tests.

The picture emerging from this study indicates that the cluttering of space with debris must be reduced in the future to minimize the collision hazard and improve spacecraft survivability. It appears that the current debris growth rates of 9 to 13% are excessive and that the associated collision hazards, particularly for larger spacecraft, will become unacceptable if these growth rates remain unchanged. There is little doubt that larger satellites and space stations will be built in the future, because of increasing demands for more and better space communications, meteorology, navigation, and remote sensing.

The question of "what solutions can be postulated" can be answered only by resolution of the space debris issues falling into three categories: 1) satellite and vehicle design, 2) operational procedures and practices, and 3) national and international policies and treaties. In the area of vehicle design the principal approaches should consider space systems for litter-free

Table 3. Summary of Present and Projected Collision Probabilities for a 1000-Day Mission

1995 (3000 objects) g_factor)	7.3	7.3	7.3	7.3
(~10000 objects) (30000 objects) (multiplying factor)	2.5	2.5	2.5	2.5
1980 (4174 objects)	10 ⁻⁶ to 10 ⁻⁵	4×10^{-5} to 4×10^{-4}	1.5×10^{-3} to 3×10^{-3}	4×10^{-2} to 8×10^{-2}
Probability of Collision with Trackable Objects	Ten meter radius spacecraft in GEO(a)	Fifty meter radius spacecraft in GEO	Ten meter radius spacecraft in LEO(b)	Fifty meter radius spacecraft in LEO

(a) GEO - Geosynchronous Orbit

(b) LEO - Low Earth Orbit

separation, reusability, retrievability, earth escape or destructive reentry into the atmosphere. The feasibility of debris collection systems should be examined.

In the operational approach and procedure areas, consideration should be given to satellite separation techniques (use of nonintersecting orbits, etc.), avoidance of crowded regions of space, and the disposal of spent satellites to "graveyard" orbits. Accidental or deliberate destruction of spacecraft should be minimized. Some of the possible approaches are summarized in Fig. 30. In short, Planet Earth appears to need a consistent and universally observed space object management policy which could ensure that future missions would be protected from unacceptable collision hazards in orbit.

SATELLITE POSITION MANAGEMENT POLICY

- 1. ESTABLISH CLOSE APPROACH MONITORING CAPABILITY
- ALERT SATELLITE USERS/OPERATORS AND OTHER APPROPRIATE AGENCIES WHENEVER PLANS ARE MADE TO REPOSITION A SATELLITE
- REDUCE NUMBER OF DEBRIS OBJECTS DURING LAUNCHING, ORBIT INJECTION AND OPERATION OF SATELLITES (e.g. SHROUDS, COVERS, PARTS JETTISONING, ETC.)
- REMOVE INACTIVE SATELLITES FROM ORBIT OR PLACE IN DISPOSAL ORBITS
- 5. TARGET BOOSTER STAGES ABOVE OR BELOW MISSION ORBIT
- 6. MINIMIZE POSSIBILITY OF ROCKET STAGE OR PAYLOAD BREAKUP OR EXPLOSION
- 7. DETERMINE EFFECTS OF ACTUAL OR PLANNED EXPLOSIONS ON THE PROBABILITY OF INADVERTENT COLLISIONS
- . USE NONINTERSECTING ORBITS
- 9. PERFORM COLLISION AVOIDANCE MANEUVERS WHEN NECESSARY

ig. 30. Operational Procedures to Minimize Collision Hazard in Space

APPENDIX

SYNCHRONOUS SATELLITE CATALOG
(28 April 1980)
ALPHABETICAL
(Ref. 3)

Company and relative solution and the same of the same of the same	7 C. 4	(1)	(Z).	,	(3) (3) (3)	(3) (4 APONEE	(3) (4) (4) (4) (4)		(4) pairs (5)
[147.9] [1.7] 4 -1.4 [167.6541]	, •	40:1ª, A 174		-1,1516	-103,9965	19324	62161	10324	0.0	
2 1,50 0 12 6.	A . A . A .		· .	4000.0-	-176,4438	19323	19129	19320	0.0	-8.8.8-
15 30 A AVEC 3	7.7.70		, ,	6460.6-	-114.0171	19324	19127	19322	0.03	€ (€ , E -
A STATE A STATE OF THE STATE OF	4.4.40	A0116.7748	۲.	1.1124	-109.0014	10323	19116	19320	0.0	8600.0-
.111.	474.	A5117,0274	-	4,4243	-149.3470	10326	19126	12321	9.78	P.8175
	4 > 4 .	40119,4275	΄.	2,05.	-105,6805	19299	19151	19297	8.34	1640,00
y by a so to asia.	4747	A011A, 75ª2	5.1	4000.	-49,2990	10354	19352	19245	6.07	6.6218
E-514 61 64 61610	4747	A0112.4966	۶.	.2.0950	89.3404	19091	19149	10003	5.84	6.1480
39324 23 9 4 475-6 3/3 (2)	G W B C .	40 94.4779	21.1	2,9214	-18.7622	18954	20237	14220	2.96	1.5600
552 4 () P 5555	, 4 B A .	AQ 71,2390	4,7,8	-0.0209	101.7170	19346	1049	18799	6.1.0	*014.3-
25 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	4.4.54	40116.4711	٠,٠	9,0465	-05.0762	19325	10324	19323	0.0	-0.6142
*6 4? 4 "SWS"A3	4 2 4 3	40116,7611	ç. .	9.0886	-127,8639	19322	19328	19322	0.0	6.477
54.54() 4 % B+	7 7 7 7	A0116, A476	~ .	0.0429	-86,9949	19326	19326	10324	0.0	* B . C . C .
84 8 SS 3 45 64	04007	A0116,7443		-0.8569	-158,9336	19393	19413	10205	3,42	-0.978-
: :. :	£35°	98 95,19Ag	21,8	-0.3278	-197,8969	19142	10159	10139	÷.	1.8643
·	€¥aC>	A0114, 12#3	٠.	1240.6-	177,0229	10124	19339	19124	4.31	2.6.04
SERVICE TO SERVICE AND SERVICES	£35'	80116. *213	۲,۰	-3,3405	108.9472	19446	19470	10179	3,90	-6.1639
	.404.	79544,4147	133.4	0.0303	161,8631	19355	19199	10305	0.0	€60€.0-
The state of the state of	404.41	A0112,0346	·.	0.7876	-125,6666	19341	19349	19248	1.86	F 0 4 +
52 . * e e. 76211	74 ° 4.	An 16.7146	103.0	-0.4907	29.7405	19126	20034	14847	1.09	72.AM1A
e yes e with a shirt	. A B A .	40 54. 1111	6.1	-0.1010	5.8701	10319	19859	19110	0.30	.1.3418
TREVE RELEASED	585	80 99.4942	¥ · • ?	-1.1932	71.9928	19443	10449	10187	2.24	0.1763
e/e 1 teers a evelt	~¶@€*	A0114,+515	•.	-1.4975	174.2120	19150	10175	10122	2.23	4,9407
13146 ** 47 4 CE ** 2	£357	Ac116.4749	~ . ~	431010-	A5.7330	19256	19461	19190	1.58	0.140
The state of the s	0,7.	43117.7716	· ·	1.4743	-43.1420	19141	19213	19154	1.54	3.4367
1 545 4 15 6 5 8 215	د ادر	40114.1546	٤٠,	**, 1475	9306.70	19347	19434	19242	16.0	0.8472
	3.	B6115.1313	۲,	Wife. C	.226.60	10277	19174	19276	0.0	.0.6105
	•	1,00001104	·	1840,75-	78.5767	32427	15478	1847	A. 0. 8. 4	13.9424

	•	•	٠		±3	1	38.000	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	10-1	الم إلا ا
	,	1	₹.	4254159-	+6.2491	11846	0 6 4 7 2	2341	48.34	21.1030
2 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	,	9110 0100	;	-11,2347	-141,3234	26011	13454	2143	47.54	11,4054
	7	43.18, 540		-34.1413	-130,0511	24645	14.20	2,00	67.37	12.6234
(-)(-)		8-46,8356		-7.4F6n	95,7489	11967	16139	1,586	2.13	12.8240
-	. 4 6 4 .	1 - 1 - 1 - 1 - 1	101.	0201.1-	122,8930	19324	19191	19317	0.17	. 9 3
	*, *,	69 54° 1346	٨٠,٢٤	3,2148	5,5215	10390	Petal	10119	0.41	-1.613-
•	. W c W ^	83105, - 4*4	1.	. n. M. 0. 1	140,3289	19323	19329	19321	0.00	C. A187
5.5 5 4 4 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5	***	40115,7170	~	21.24.27	-110.1940	19324	19129	19323	3.91	# (C to) () =
() in unit () experience of the experience of th	5,77	ACT16.7919	٠٠٢	+9.74.5+	-135.0284	16397	19320	19320	0.0	-7.6718
-	583	2767.81624	211,1	4.58.6	120.0362	20793	25018	19663	10.09	0.0861
•	5 8 8 9	A0114.4516	ij	73.1943	14,1944	19319	19335	14317	0.21	Bech.o
m partition of profession	582	A0105.7848	12.7	0.0568	93,1007	19290	19350	19247	96.0	.0.0006
147	C 4 8 C 7	Mg112,0943	c: *	0.4916	116.1430	19543	1961	19535	96.0	-5.7199
•	SAGO.	40117.7125	1	7, 1956	*51,4358	18349	18694	14104	10.01	22.5278
e/e el** a5), *?	74.00V	80117,2044	۵.	-0.6419	-158,7033	19091	18150	17972	5.64	32.4189
85	24.65	AD 93, 4433	29.2	-4,2996	24,4297	1,893	16928	14196	80.0	19,1939
F 43 A 1+1E-SAT 1+ F1	148,00	A0116.4590	٠,	-0.0842	*18.5261	19325	19356	19324	0.0	+0.ñi2n
84 71 148 3150 W 14 W	7245AT	90117.9253	7:	1050.0-	173,6352	19324	19338	10319	0.10	0.0017
I got at S Bar. To a?	NERRO	19 49, #250	434,2	6.04.0	-147,2232	19309	19396	19265	11,41	-6.4224
49 11 4 1476.547 3 43	13×5×5	A0 14.6575	104.1	6, 4, 4	71.9265	19313	19338	19309	6.30	2. ñ 1 0 4
•	c¶αĉ≯	A3 85,8722	31.1	4.0107 -	145,8998	19579	10875	1 4 1 4 1	9.20	7.1684
2 9 1 4 5 3 to 1 4 5 4	45,000	A3116.4376	2.4	1.4913	-1.6900	19328	1912	19317	2.30	0.6382
•	145	A0116.46"8	٠.	-9.9117	-21.3250	19329	10335	19321	0.0	-0.42.4
1,	**S.2.	#gf14.9248	3.1	-0.7549	178.8015	19319	19138	19369	6.20	-6.6616
	19 S m C u	49112,2577	;	7.8948	37,9165	19397	19329	19320	0.0	0. ñ349
4 4 5 5 1 4 5 5 4 5 5 5 5 5 5 5 5 5 5 5	* * S	43112.2619	•	7.0982	56,3892	19321	19116	19322	0.24	9.014
Tager (#5 By) . F To as	45,000	6217,4117	·:	4.40.0	-24,4603	19341	19131	10318	0.0	-5.0450
2* yes . 15 3.2.	3	97116.4642	٠,٠	P & C . C -	-27,5265	19357	19127	19323	0.01	
1 - 4 P - 2 P - 2 P - 4 P - 4	***	40112,0515	`.	5,424	43,5251	19326	1935	19321	9.13	****

SYNTHOUS STALING FOR DATERBILLD. (28	6 744	., 6 .,	÷	A	30,116,0	7 7 7 10 10 10 10 10 10 10 10 10 10 10 10 10	#+00 4	pc. 1, r.t.	Ž	* J - 82 ·
19374 TY 41 A TATE SAT 4AFM4	748×C-	A3111.5177	·.	-1.1(.1	***, 9419	19326	.2.61	19323	φ. Ε) Ε'	
7 11 9.	F0~541	A0112,2412	۷.	0.08.0	63.0271	19355	19126	1 0 12 3	0.01	
•	4 7 4 7	40116.0340	0	.4,9924	-34,9499	16646	34435	0 4 6 7 1	24.24	1000
6.	841 (3)	A0112. A313	4.2	5,5476	-107.9217	16323	19192	10318	6.1	•6.5.3•
7€577 ▲	04.04	40106.5504	17,4	.4.6850	56.0171	19224	19126	1.9966	e a .	• • • • •
39746 76 25 A LES-R	LC . AB	A0117. A033	1.3	22.4908	-105,2996	19319	19128	10310	25.24	0.4.0
15 25 €	2 404 40	80119.7607	1.2	24,7514	-146.6339	10043	10036	103.2	25.40	-7.3334
09747 "4 24 B LES-0	רכי רשט	AF119,6572	.,	17.0637	.03.4104	10335	19338	19317	25.24	0.671
09637 76 17 A MADICAT &	4843	AD114,4542	•	0.8983	-15,2295	10319	10334	10319	0.0	-5.6724
APIGAM A 12 84	ASAU	90119.9141	3.1	-0.2629	176.4200	10320	10358	19320	86.5	\$ K K G * C +
4-4-4-7-04-10 4 10-10-10-10-10-10-10-10-10-10-10-10-10-1	4845	A0112.1136	6.4	-0. 4803	72.6184	16861	16236	10319	1,42	-0.6167
77109 A METENS	# S #	A0106,4348	17.4	9067.0	0.7830	19367	19138	19262	0.92	0064.0
74 65 A HO, 417	NORAD	74210.1745	2093,A	*2,5968	114,7869	19349	19359	10345	0.0	46.9454
4111 Cf47 4 78 67 81880	C + 42	AD 85.4129	31.4	-0.45R2	-18.3601	19317	19334	19316	1.10	-6.7166
34393 47 21 A VATO 1	C . 4 .	40100.9276	14.1	3,2554	-105.9080	19313	16437	19313	4.93	01,0'0-
04932 71-37 A 1473 2	C * 4 7	AB 24.1370	6.16	-1,6820	141.3117	19327	19128	10001	3.0	2.9404
09745 47 4 A VATO 3-8	6.44	A1112, 17A9	4.6	-0.7719	-60.0309	19459	20249	1 - 3 - 0	1.00	6.711
IIIIS PEIOS A VATO-IIIC	C. V'4	A0113, 1948	÷.	-2,4855	-50,0771	19310	10.61	19310	3.30	10.0.0.
09891 72"17 A 008 1570	پ ٽ	83113.1195	c. •	-3.2396	-162.9695	19320	19152	10284	6.02	*0.108#
81524 TEST A TOS 1573 BJB (2)	40343	A0117, 45 2	1.1	4,7808	-106,7643	19123	10436	19191	2 6 '6	1.4414
2112 Sec 9 65 92 91660	€2,	A0115,9144	4.2	2,6843	-49,3091	19329	10310	10313	2.87	9714.0-
(2) 8/6 2112 Sec C 66 9. 91660	* Vac*	40111,0378	7.1	1,7871	-102,0189	19292	1931	19240	2.89	1.144
03833 77, 7 A "SC 1281	scr	90111,4482	۲.۴	-1.0357	69,2972	19331	1623	10351	2.39	6+10.0-
33855 *7 * C "PS 3151 A/B (2)	4002	Antifa. Pico	۲.٠	2.1987	-70.0822	19354	19356	19279	2.38	0.6027
Seta Sec. e. alls. 2000	-1384°	A0112,44A5	\$.	2.1400	-3,1395	19333	19758	19225	3.31	
095(6 3: 145 B.d (2)	€406.	A01:6, 1270	2.7	Q.A.	103,9079	19299	19150	19218	3.37	4 + M . O
The sec a coll acto	\$ C.	A0134.1547	17.6	4 c 16 4 . 15 4	74,4466	19328	10118	10310	9	+6.4074
(2) 8/6 118x sec. 4 3 81818	C400.	49 77, 4477	•	* 4.7824	61.9940	19548	1001	10028	7.03	4 5 4 F . C
0 to 2 to 10 to 2 to	C 4 0 C 7	A0117.7211	÷	4. £ \$ 1. ª	-78,852-	18699	1019	14847	10.34	11.7721

rect pairs	10.35 71.8449	9,36 0.n459	5,29 -2,1294	1,17 -0,8147	2,00 0.6302	2,54 70,6367	1,24 0.0481	1,29 "3,658#	9.24 26.0717	7.53 24.4869	4.14 0.8745	4.33 011.0274	4.02 -0.1767	2.59 -0.2835	2.98 -18.7977	2.68 -0.0634	0.61 1.4904	0.44 *16.4404	0.47 -1.0454	1.47 D.A745	1.69 26.1467	4.48 .0.000	2.19 0.0027	2.04 617.4545	2.00 .0.004	9.03	9.00 4.5244	6.68 -0.6437	3,59 -0.0222
pekture	11855	10201	10317	19315	10274	19298	19356	10377	14135	14241	19308	19465	19323	19326	10405	19291	19171	19321	10331	19391	19466	19323	10318	19336	10324	10211	46241	10324	14613
y:tCav	19633	10149	19528	19114	19173	19154	19329	1995	18334	10479	10338	20138	19339	19319	20405	19354	19199	11602	19494	19356	19100	19329	19338	2087	19326	1017	20176	1017	22539
· · · · · · · · · · · · · · · · · · ·	14956	19310	19317	19321	19354	10314	19393	19469	18378	15367	19318	10407	19310	19345	\$6704	19291	19321	19497	10311	19326	18243	19329	19326	19822	19323	19348	19900	10317	21919
JAST TUBE	2.3035	-09.9254	199,7477	-99.2481	72.353A	72,6881	-132,7876	2,9221	102.9498	* 40 . D & 6 R	-107.4065	-31.4969	-177.3396	-90.2343	105.0999	18.6491	104.6727	-66.8194	-143,8296	-135.0192	72.1443	175.4629	-12.3631	-46.6962	65.6391	3.9424	-53.3667	80.9985	74.4051
	7.5747	4,1447	1,0943	1950.0-	-0.146	15\$0.5	9.4476	-1,1949	-9.2656	6010.8	2.1339	3.8376	-1.8466	1.6671	1464.5-	-0.4300	9.4156	-3.22.6-	-0.3974	-0.0862	0,7896	1.1417	-2.5494	-1.9429	1.4921	7.0117	4,36.3	9+11.4-	-2,5(42
•	ć. -	;	0.0	۲.	14.3	``	:	۶.٥	754.7	787.0	1.1	r.	9.6	2.1	°:	14.5	•	1.6	3.2	7 . 4	227.0	:	٠.		13.7	;	545.3	;	•••
t, D	6114, 4, 11, 4	A3111, A716	A3118.1524	A3114, 1729	46107,591	A0116, 1541	#0114.46P1	A0112.1346	19229.2742	78 65-noño	A0117.4976	80112.4995	90118.ñ799	80116.8918	89114.1072	80104.5110	83114-1416	80118.431	A9113.79A2	93116.5344	19251.4291	49117.6414	40113.2375	93118.2559	A010A.A293	1.61.21174	19741-1345	AC111. 1915	1414.4155 e
- σ - - - - - -	: # # €	, 20 10 10 10 10 10 10 10 10 10 10 10 10 10	. ♥ e0'	30%	3 0	و د و	در	24.9€.v	NERAD	NORAD	04 80 2	04 40 2	2740%	SCF	04 4 G V	\$C.	۵۵۶	2 V a C Y	₹ 0 ₽	SOF	10840	کائر	کائو	1,004,0	کات	₹54	. ▼65.	1213.ES	58 CUN
ATT ACTOR STATE OF THE LOCAL			ņ	1014 201	ř	000	٥¢.	(2) 8/8 78 4 Sec. 8	(£1 #8501) 9484 54. 3	192 8347 (105CS 26)	1840 Sec. 4	4 205 9431+2 R/8 2	2340 Sec. 8	£ \$ \$ \$ \$ \$ \$ \$		48 40 5c 8	4 755 9417	(2) 8/8 8886/4886 Sec. 2	Bird Sec. 6	Took Sec 1	12) A/F 2 4 4 1 5 5 1 A / B (5)	2000 sec 8	A 395 3463	Be seestsee ser 2	***¢ >c. f	E T U M F T	*****	4 24 4 2 4 4	THIS TO CALL BARGE BE
,	5	,					16 6.			. 6 44	66 1.				:;	£ \$134			31 W K A	. 1114	0 1116.	9:1:6.	4 40 00		6 80 04	* * * * * * * * * * * * * * * * * * * *	1 1 1 1 1	7 5 6 F	• :
\$ *** ** \$											28887	8.5.5				2 4 9 7 4 9			10001	11114		. 61111	11671	11685	11622		- CE 51 0	23339 7	23842

512.21	Secretary Signature	34TE=90119, (28	APH BC	С.	ŗ	1 4 1 1 6	adolitics:	E - 0	4.004	PERIGRE	ואכר	D. i.r.
		BICSIBLE 9473 EARANT	466354	79243.1359	243.9	-1.7463	75.6732	19410	19458	19196	1.73	-0.0644
	* 60 %	46,746	688 ⁽¹⁾	19211.1742	272.9	15,2127	-107,5282	14929	24133	14463	2.37	0.4284
		400046	A GO A P	P0113,7301		2,7167	-31,1658	19261	19322	10201	2.93	0.7824
			34667	A0115.A170	5.2	0,6949	-107,0916	19645	19791	19682	1.64	-9.1250
		•	04 40V	80113,5403	٠.	0,3621	-101.7443	19729	10110	10788	9.0	-9.7102
	\$ 0 T & 4	34 6	5.8 S.C	A0113, 3184	5.7	-0.1912	34,6614	10297	10116	10277	0,29	0.6109
	10 14 0	EG 4 €	UVON	80115.8642		0,3600	166.0661	19735	10954	19758	0.31	.9.455
			455°	80 40,6648	1.60	-0,3464	39, 3703	19400	10410	19228	1.1	6.42.2
	4 77 64	4445,744	5.5°	86 63,4846	55.8	0.1628	39.8280	19398	19338	19248	1.03	0.003
	4 38 6.	941,74+5	5850	A0108.1947	13.6	-0.0805	63,2303	19330	10338	10323	9.29	-0,5485
	19:13	4.4.9	404	\$9232,4391	251.6	-0,3912	.77,3870	19321	19396	19245	60.0	0.1983
	* 6 2 9 *	α * * · · · · · · · · · · · · · · · · ·	₹ 0 €	80116,7364	۲.۲	-0.0173	-118,9841	19323	10154	14372	0.0	-6.6619
	* * *	# 1 	3 08	80111,3605	٧٠,	-6.5039	-76,0661	18084	23396	14845	6.67	9.6079
	1 4 4	· lels	1145	19847, 1272	139.7	-0,6859	-170,6353	19950	2000	18674	1.59	.0.1961
	¥ 10164	SAYVETA	58 B21		14.1	4,7816	-105,9416	19594	10801	19091	5.78	0.0100
	* * * * * * * * * * * * * * * * * * * *	SKYVET 24	18 391	80 68,64ñ7	53.4	-0.4633	42.0761	19077	19932	10016	1.71	1.5421
	4 34	7.048	4 7 7	80111.0546	٥.	-1,2462	-139,6118	19318	95167	10316	69.	-0.910
		E • U • S	484	80116.4131	₹.	0.3948	.74,9336	19394	19336	19270	9.54	-0.1672
	¥ .016.	(# S#CE)C+S+S	4847	80100,0610	14.0	9.5459	.88.6389	19436	10438	10101	1.09	-0.6-11
	* 101**	4 - M - 2 C 1 G 1 - 5	FRACE	80108.7387	19.1	1.1255	-12.0090	19321	10112	19318	1.23	-0.8103
	* *	8 - H 1 - C 1 6 1 + 5	FRAMCE	A0108.7094	11,3	1.4901	-11,5187	19318	10139	19298	1.74	-0.8099
91919		I AEU AF PC MY	2460.⊀	80:08.4095	13.6	-0.7744	44.0699	10317	19338	10314	2,1	·0.ñ275
88588		44 555 JS64 84	64 003	AD 54.6272	*.	-2.4813	73,5285	14041	2011	14549	. 43	100 i. D
63527		44 00\$ 4240,4293,4932	√اعو ₹ن	40 65.1778	54.8	-5.9270	-113.4702	1921	40101	19271	3,78	-0.444
8 9 8 6 E		21691862914829 806 35	[4 e C 2	A 9116.7309	۲۰,	5,9963	-113.7416	19248	10111	19246	2.91	.0.5.0.
81213		2 \$ (2 4 2	_¥aC*	89135.42 ⁴ 3	11.	-0.0455	10.1234	19503	*****	10.43	8 0.0	0.6347
83538			900	A3 11.1216	107.9	-1.9[11	84,0793	19213	10473	14045	7.67	1.4407
		> 1 (2 v ·	6.0	39347,4416	241.4	-A.54AB	44.4004	82448	70478	1 40 40	7,73	.9.416A
91518		7 8 6 7 8 7		#0112.5541	;	21417	16.1945	10:28		10152	1.67	¢ 0 • ¢ · č

S * * ? * ?	. 5-2-5		COMPANY FOR TENTIONERS FOR TOTALED STORESTAND	(Lib hat ti.	, , ,	;	I ATTY I DE	EP274 TY LATITUE LONGITUDE HEIGHT APONEF PEHIGEE INCL DRIFT	16164T	APODER	PFHIGEE	אנר באנה	F 1 8 2
9 9 9 9 0			***	G C			1.855	-86,1843	19190	1954	19167	2,43	2,48 -0.ñ774
81918			* * * * * * * * * * * * * * * * * * * *	_9e_7	5.5 And \$111, 4.7	•	0.0054	0.0054 -134,8904 19246 19414 19234	19218	19414	19234	0.10	0.10 0.0104
81573	16511	•	** ****	9	1.047 Ag 40,4516 64.3	63.3		-1.132n A1.1239	19391	19672	14962	2.30	2,30 n.185A
11619	, sec	•	** ** ** ** ** ** ** ** ** ** ** ** **	a	1,94 394,2941 289.7	7.685		-0.0813 -138.8344	80808	20908 21158	19697	1.19	1.15 -3.7189
3 6 2 4 0	74 27 4	-	4 84 5 F	* C	AEST UN POSSONNIZ 2.3	7.3	-0.0890	-0.0490 -99.0691	19325	19328	19322	0.0	0.04 -0.Aj27
11494 49 27	4.	_	1) 74 by 2 +	7, .,4*	F. C. 40110.6587 2.4	٠.٠	-9.0845	-9.0845 -90.9486	19326	19326 19388	19321	0.02	0.02 -0.6/98
964.0		•	* C * & * * * * *	3, 10 j.	46 CT UN BOSSB,7445 0.3	3.3	3.5546	0.0846 -123,4959	19324	19324 19329	19322	0.03	0.03 -0.ABZ4
1423			Horasta William	(4	1, 247 A3 24,4923 93.1	93.1	0.4915	0.4915 116.0599	19540	19540 19606	19931	6.74	0.75 -4.932
3.691			4 3 x 2 ;	CARBOLA	KIBRAS #0117,8560 1.1	1.1	0.1414	0,1414 -171,6690	19711	19741 19932	19723	0.21	0.21 -0.1734
6116			b 3 * £ .	describe	NJCHAD #3114,9242 3.1	1.1	0.4942	0.4942 -104.2659 19348 19408	19348	19498	10324	1,51	1,51 -0.8014

Notes:

- 1) Date of the orbit element set (year, day number, fraction of day)
- Time since epoch of element set (days)

?

- 3) Position of object on 28 April 1980 at 0000 Z (deg N, deg E, nmi)
- 4) Orbital parameters for epoch data (nmi, nmi, deg)
- Longitude drift rate for epoch date (deg/day East) Westerly (negating) drift for mean motion n < 1.002738 revolutions/day. ()

REFLRENCES

- 1. V.A. Chobotov, "On The Probability of Satellite Collisions in Earth Orbits," TOR-0079(4071-07)-1, The Aerospace Corporation, El Segundo, CA June 21, 1979.
- 2. D.J. Kessler, P.M. Landry, B.G. Cour-Palais, R.E. Taylor, "Collision Avoidance in Space," IEEE Spectrum, June 1980.
- 3. R.A. Marsh, Private Communication.
- 4. Revised Table of Earth Satellites, Royal Aircraft Establishment Farnborough, Hauts., England, Vol. 2 (1969-1973).
- 5. W.A. Feess, "Reconstruction of Skynet #2(B) Apogee Burn Performance," TOR-0059(6112-10)-4, The Aerospace Corporation. El Segundo, CA, February 1971.
- 6. L.J. Tedeschi, "Analysis of Doppler Data in Support of the Skynet 2 Failure Investigation," TOR-0059(6112-10)-3, The Aerospace Corporation, El Segundo, CA.
- 7. G. Wrenn, "Geos 2 in Space Collision?," Nature Vol. 274, 17 August 1978.
- 8. D.J. Kessler and B.G. Cour-Palais, "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt," J. Geophysical Res., Vol. 83, No. A6, June 1, 1978.
- 9. W.L. Morgan, "Geosynchronous Satellite Log," COMSAT Technical Review, Vol. 8 No. 1, Spring 1978, pp. 219-237.
- 10. W.L. Morgan, "Satellite Utilization of the Geostationary Orbit," COMSAT Technical Review, Vol. 6, No. 1, Spring 1976, pp. 195-205.
- 11. D.H. Martin, Communication Satellites 1958 to 1982," TR-79-078, The Aerospace Corporation, El Segundo, CA, September 10, 1979.
- 12. Physical Nature and Technical Attributes of the Geostationary Orbit Study Prepared by the Secre ariat of the United Nations, A/AC. 105/203 29 August 1977.
- E.J. Opik, Interplanetary Encounters (Close-Range Gravitational Interactions), Elsevier Scientific Publishing Co. New York, 1976.
- 14. T.R. Gurlitz "Program MINRNG," TOR-0080(5409-49)-1, The Aerospace Corporation, El Segundo, CA, May 7, 1980.

- 15. Collision Probability of the Apollo Spacecraft with Objects in Earth Orbit, NASA MSC Internal Note No. 67-FM-44, April 10, 1967.
- 16. M.R. Spiegel, <u>Probability and Statistics</u>, Shaum's Outline Series, McGraw-Hill Book Co., <u>September 1975</u>.

